Reduction of the spin-orbit potential in light drip-line nuclei

G.A. Lalazissis, D. Vretenar[1], W. Pöschl and P. Ring
Physik-Department der Technischen Universität München,
D-85747 Garching, Germany

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Abstract

The isospin dependence of the spin-orbit interaction in light neutron rich nuclei is investigated in the framework of relativistic mean field theory. The magnitude of the spin-orbit potential is considerably reduced in drip line nuclei, resulting in smaller energy splittings between spin-orbit partners. The effect does not depend on the parametrization of the effective Lagrangian. The results are compared with corresponding calculations in the non-relativistic Skyrme model.

The spin-orbit interaction plays a central role in the physics of nuclear structure. It is rooted in the basis of the nuclear shell model, where its inclusion is essential in order to reproduce the experimentally established magic numbers. In non-relativistic models based on the mean field approximation, the spin-orbit potential is included in a phenomenological way. Of course such an ansatz introduces an additional parameter, the strength of the spin-orbit interaction. The value of this parameter is usually adjusted to the experimental spin-orbit splittings in spherical nuclei, for example ^{16}O. On the other hand, in the relativistic framework the nucleons are described as Dirac spinors. This means that in the relativistic description of the nuclear many-body problem, the spin-orbit interaction arises naturally from the Dirac-Lorenz structure of the effective Lagrangian. No additional strength parameter is necessary, and relativistic models reproduce the empirical spin-orbit splittings.

Many properties of nuclei along the line of beta stability have been successfully described in the framework of models based on the mean-field approximation. Conventional non-relativistic models that include density dependent interactions with finite range (Gogny) [1], or zero-range (Skyrme)

[1]Alexander von Humboldt Fellow, on leave of absence from University of Zagreb, Croatia

forces [2], have been extensively used to describe the structure of stable nuclei. More recently, it has been shown that models based on the relativistic mean-field theory [3, 4] provide an elegant and economical framework, in which properties of nuclear matter and finite nuclei, as well as the dynamics of heavy-ion collisions, can be calculated (for a recent review see [5]). In comparison with conventional non-relativistic approaches, relativistic models explicitly include mesonic degrees of freedom and describe the nucleons as Dirac particles. Non-relativistic models and the relativistic mean-field theory predict very similar results for many properties of beta stable nuclei. However, cases have been found where the non-relativistic description of nuclear structure fails. An example is the anomalous kink in the isotope shifts of Pb nuclei [6]. This phenomenon could not be explained neither by the Skyrme model, nor by the Gogny approach. Nevertheless, it is reproduced very naturally in relativistic mean-field calculations. A more careful analysis [7] has shown that the origin of this discrepancy is the isospin dependence of the spin-orbit term. With a spin-orbit term modified in such a way that it is similar to that derived in the relativistic mean-field model, the Skyrme model produces comparable results for the isotope shifts [8]. Recently, in Ref [9] another modification in the spin-orbit term of the energy functional of the Skyrme model has been proposed. An additional parameter was introduced, which is adjusted to reproduce the kink in the isotope shifts. Compared to conventional Skyrme forces, this approach produces a very different isospin dependence of the spin-orbit potential.

Experiments with radioctive nuclear beams provide the opportunity to study nuclei with large neutron excess. Neutron drip lines of relatively light nuclei have become accesible, and the investigation of properties of such exotic objects is becoming one of the most exciting challenges in the physics of nuclear structure. Because of their relevance to the r-process in nucleosynthesis, nuclei close to the neutron drip line are also very important in nuclear astrophysics. Knowledge of their stucture and properties would help the determination of astrophysical conditions for the formation of neutron rich stable isotopes [10]. For drip line isotopes, nuclear shell effects become very important and the spin-orbit term plays an essential role. Very different scenarios for the isospin dependence of the spin-orbit interaction have been suggested in the Skyrme model [11], and in the relativistic mean-field theory [12]. In the present work we investigate the behaviour of the spin-orbit potential in light neutron rich nuclei. The description of drip line nuclei is complicated by the closeness of the Fermi level to continuum states. Pairing correlations, and the coupling between bound states and positive energy particle continuum, are described in the Relativistic Hartree-Bogoliubov (RHB) model in coordinate space.

In the Hartree approximation for the self-consistent mean field, the RHB

equations read

$$\begin{pmatrix} \hat{h}_D - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D + m + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}. \tag{1}$$

where $\hat{h}_D$ is the single-nucleon Dirac hamiltonian, and m is the nucleon mass. U_k and V_k are quasi-particle Dirac spinors, and E_k denotes the quasi-particle energies. The RHB equations are non-linear integro-differential equations. They have to be solved self-consistently, with potentials determined in the mean-field approximation from solutions of Klein-Gordon equations for mesons. In the particle-particle (pp) channel the pairing interaction is approximated by a two-body finite range Gogny interaction

$$V^{pp}(1,2) \;=\; \sum_{1,2} e^{-(\mathbf{r}_1 - \mathbf{r}_2/\mu_i)^2} \,(W_i \;+\; B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau), \tag{2}$$

with parameters μ_i, W_i, B_i, H_i and M_i ($i = 1, 2$).

In the present investigation we consider the isotopic chains of the even-even Ne and Mg nuclei. Systematic RHB calculations of ground state properties have been performed, and the results reveal many interesting features. In this report we focus on the behaviour of the spin-orbit term. In the relativistic mean-field approximation, the spin-orbit potential originates from the addition of two large fields: the field of the vector mesons (short range repulsion), and the scalar field of the sigma meson (intermediate attraction). In the first order approximation, and assuming spherical symmetry, the spin orbit term can be written as

$$V_{s.o.} = \frac{1}{r}\frac{\partial}{\partial r} V_{ls}(r), \tag{3}$$

where V_{ls} is the spin-orbit potential [5, 13]

$$V_{ls} = \frac{m}{m_{eff}}(V - S). \tag{4}$$

V and S denote the repulsive vector and the attractive scalar potentials, respectively. m_{eff} is the effective mass

$$m_{eff} = m - \frac{1}{2}(V - S). \tag{5}$$

In the following we present results for the even-even Ne and Mg isotopes. For the mean-field Lagrangian the NL3 [14] parametrization has been used, and the parameter set D1S [15] for the finite range pairing interaction. Using the vector and scalar potentials from the self-consistent ground-state solutions, we have computed from (3) - (5) the corresponding spin-orbit terms for Ne and Mg chains. They are displayed in the upper panels of Fig. 1, as

function of the radial distance from the center of the nucleus. The magnitude of the spin-orbit term $V_{s.o.}$ decreases as we add more neutrons, i.e. more units of isospin. The reduction for nuclei close to the neutron drip is $\approx 40\%$ in the surface region, as compared to values which corespond to beta stable nuclei. This implies a significant weakening of the spin-orbit interaction. The minimum of $V_{s.o.}$ is also shifted outwards, and this indicates the large spatial extension of the scalar and vector densities, which become very diffused on the surface. The main contribution to the densities in this region comes from the outermost neutron orbitals, whose wave functions are extened in space. The same effect can be observed in Fig. 2, where the *rms* radii are plotted as function of neutron number for Ne (a), and Mg (b) isotopes. We display neutron and proton *rms* radii, and the $N^{1/3}$ curves. The dashed curves are normalized so that they coincide with neutron *rms* radii for ^{20}Ne and ^{24}Mg, respectively. Close to the neutron drip a sharp increase of neutron radii is observed, as compared to the mean-field curves $N^{1/3}$. This sudden increase indicates a possible formation of multi-neutron halo [16].

In Fig. 3 we display the spin-orbit splittings of the neutron levels

$$E_{ls} = E_{n,l,j=l-1/2} - E_{n,l,j=l+1/2}, \tag{6}$$

for Ne and Mg isotopes as function of the neutron number. The neutron spin-orbit splittings decrease with neutron number. This is consistent with the gradual weakening of the spin-orbit term that is shown in Fig. 1. However, the result is at variance with non-relativistic mean-field studies that have used Skyrme forces [11]. In the Skyrme model, the spin-orbit term included in the self-consistent mean field is of the form

$$V_\tau^{ls} = \mathbf{W}_\tau(\mathbf{r})(\mathbf{p} \times \sigma), \tag{7}$$

with

$$\mathbf{W}_\tau(\mathbf{r}) = W_1 \nabla \rho_\tau + W_2 \nabla \rho_{\tau' \neq \tau} \tag{8}$$

where ρ_τ is the density for neutrons or protons ($\tau = n$ or p) and τ' is the opposite isospin. W_1 and W_2 are parameters. In the non-relativistic reduction of the Dirac equation the resulting spin orbit term takes the same form. However, the corresponding quantities W_1 and W_2 are coordinate dependent, and also introduce an isospin dependence which has its origin in the coupling constant g_ρ of the ρ-meson [7]

$$\begin{aligned} W_1(r) &= \frac{1}{4m^2 m^{*2}}(C_\sigma^2 + C_\omega^2 + C_\rho^2) \\ W_2(r) &= \frac{1}{4m^2 m^{*2}}(C_\sigma^2 + C_\omega^2 - C_\rho^2). \end{aligned} \tag{9}$$

with $C_i^2 = (mg_i/m_i)^2$ for $i = \sigma, \omega, \rho$. Non linear self-interaction terms for the sigma meson are included in the derivation of W_1 and W_2.

With the self-consistent proton and neutron densities of the drip line nucleus ^{40}Ne, we have computed the neutron spin-orbit terms both in the Skyrme model and in the relativistic mean-field model. Spherical symmetry is assumed, and for simplicity, pairing has been neglected in this illustrative calculations. The effective force Skp [17] has been used in Skyrme calculations, and the NL3 parameter set for the effective relativistic Lagrangian. The results are summarized in Fig. 4. In Figs. 4a,b,d, and e, the neutron and proton densities, and the corresponding gradients are shown. The spin-orbit term calculated with the Skyrme force Skp is displayed in Fig. 4g (solid line), and the one resulting from relativistic calculations in Fig. 4i. The minimum in the surface region is for the Skyrme force Skp $\approx 40\%$ deeper than in the relativistic calculations. In order to have a more complete comparison, in Fig. 4f we also display results of the relativistic model with the NL1 [18, 19] (dotted line), and NL-SH [20] (dashed line) effective forces. They turn out to be quite similar to those obtained with NL3. The small radial shift between the two minima is caused by the different isospin properties of the two parameter sets. The key point, however, is that in all relativistic calculations the spin-orbit term is much weaker compared to results of the Skyrme model. This of course explains the difference in the calculated spin-obit splittings. The spin-orbit term in the Skyrme model does not decrease in magnitude even for extremely neutron rich nuclei. This results in large spin-obit splittings and quenching of the shell effects.

The Skyrme model uses in principle an isospin independent two-body spin-orbit force. The exchange term to this force, however, induces a strong isospin dependence for the spin-orbit term in the self-consistent mean field. The resulting ratio for the W_i parameters is: W_1/W_2=2. On the other hand, in the relativistic approach the spin-orbit term is to a large extent a pure single-particle effect, as can be shown by a non-relativistic reduction of the Dirac equation. This is also the case in relativistic Hartree-Fock calculations. Therefore in the relativistic description there is no contribution from a two-body spin-orbit interaction, and consequently no exchange term. The isospin dependence of the parameters W_1 and W_2 in the relativistic theory comes from the ρ-meson. The contribution of the ρ mean-field increases with neutron number. This effect is illustrated in the lower panel in Fig. 1, where we plot the self-consistent ρ-meson potential for Ne and Mg isotopes. However, its contribution is much smaller than that of the σ and ω fields, and therefore the isospin dependence of W_1 and W_2 should be rather weak. In Figure 4c we display the relativistic W_1 and W_2 calculated with the effective force NL3. The radial dependence comes from the effective mass m^*. The two curves are very similar and in the surface region, where the spin-orbit term is peaked, their ratio is approximately $W_1/W_2 \approx 1.1$. If we now use the same ratio in the Skyrme model, the resulting spin-orbit term for the Skp force is displayed in Fig. 4g (dotted line). A significant reduction of the spin-orbit

term is observed, and the results are comparable to those obtained in the relativistic calculations. We have also computed the spin-orbit term for the effective interaction SkM* [21] and the two values of the ratio W_1/W_2. The results displayed in Fig. 4h show the same reduction in the surface region.

In conclusion, we have shown that, in the framework of relativistic mean field theory, the magnitude of the spin-orbit potential is considerably reduced in light drip line nuclei. With the increase of the neutron number, the effective one-body spin-orbit interaction becomes weaker. This result in a reduction of the energy splittings between spin-orbit partners. The reduction of the spin-orbit potential is especially pronounced in the surface region, and does not depend on a particular parameter set used for the effective Lagrangian. These results are at variance with those calculated with the non-relativistic Skyrme model. It has been shown that the differences have their origin in the isospin dependence of the spin-orbit terms in the two models. If the spin-orbit term of the Skyrme model is modified in such a way that it does not depend so strongly on the isospin, the reduction of the spin-orbit potential is comparable to that observed in relativistic mean-field calculations.

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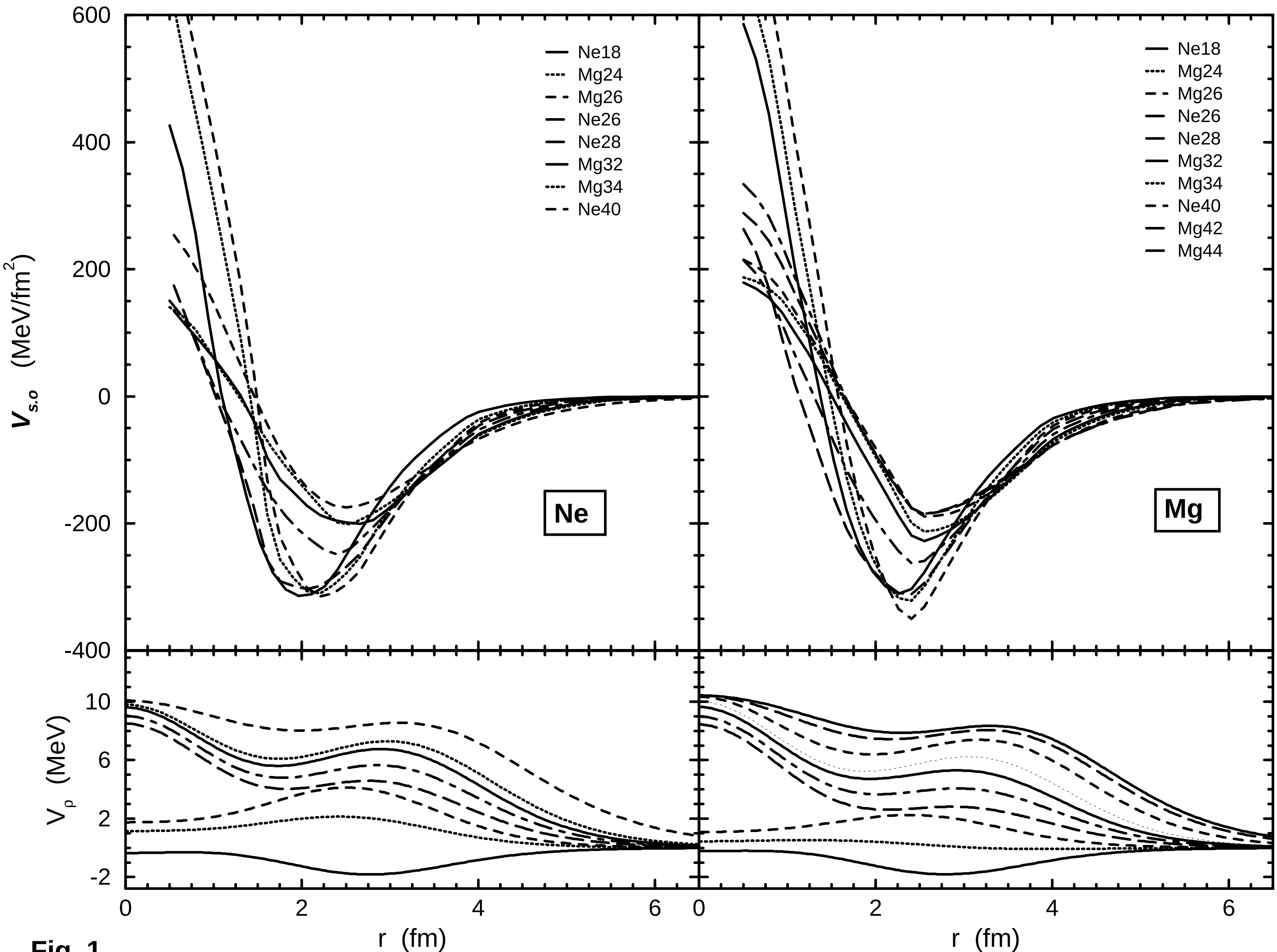
Figure Captions

- **Fig. 1** Radial dependence of the spin-orbit potential in self-consistent solutions for the ground-states of Ne (a), and Mg (b) isotopes. In the lower panel the self-consistent ρ-meson potentials are displayed. The NL3 parametrization has been used for the mean-field Lagrangian, and the parameter set D1S for the pairing interaction.
- **Fig. 2** Calculated *rms* radii for Ne (a), and Mg (b) isotopes as functions of neutron number.
- **Fig. 3** Energy splittings between spin-orbit partners, for neutron levels in Ne and Mg nuclei, as functions of neutron number.
- **Fig. 4** Self-consistent proton and neutron densities, the corresponding gradients, and the spin-orbit terms for ^{40}Ne. Results are displayed for the relativistic mean-field model and non-relativistic Skyrme calculations. For description, see text.



Ne18
Mg24
Mg26
Ne26
Ne28
Mg32
Mg34
Ne40
Mg42
Mg44
Ne
Mg
$V_{s.o}$ (MeV/fm^2)
V_ρ (MeV)
r (fm)
600
400
200
0
-200
-400
10
6
2
-2
0
2
4
6

Fig. 1

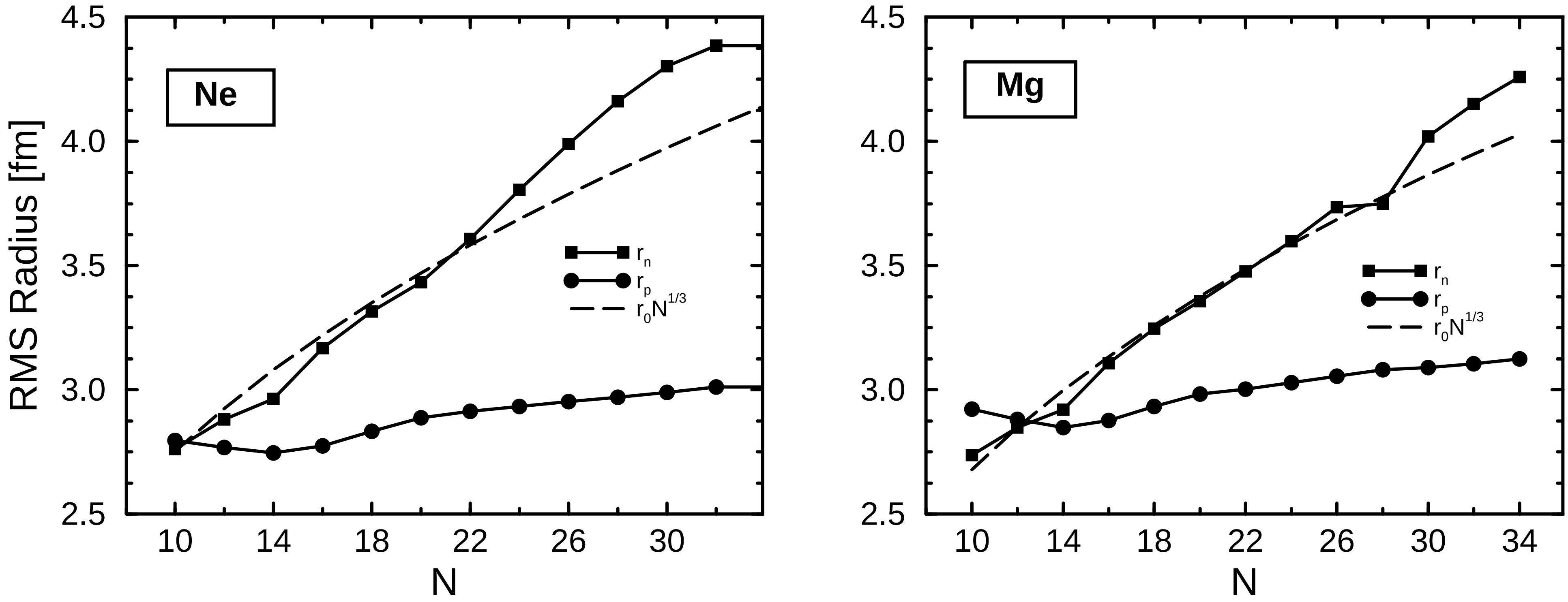
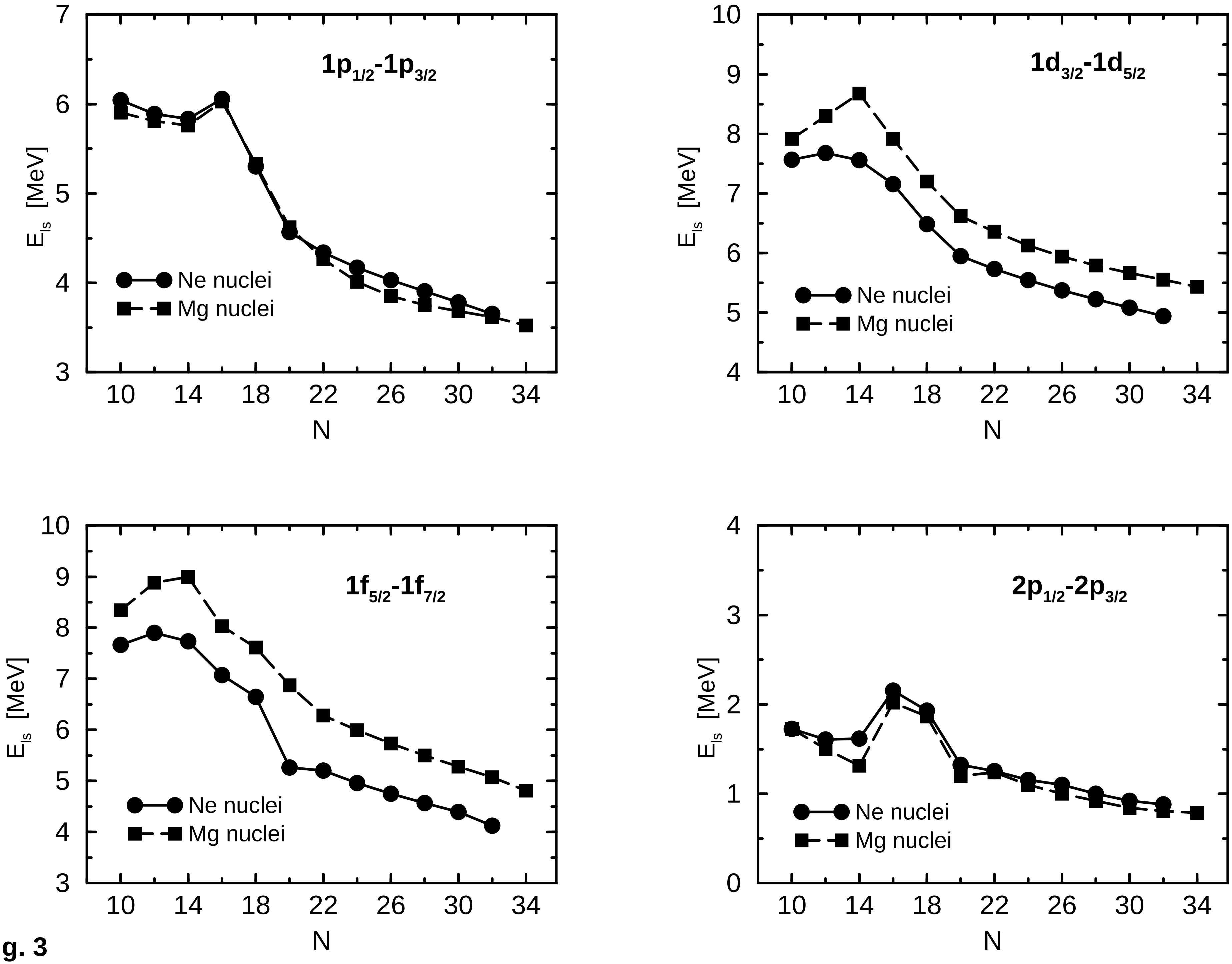
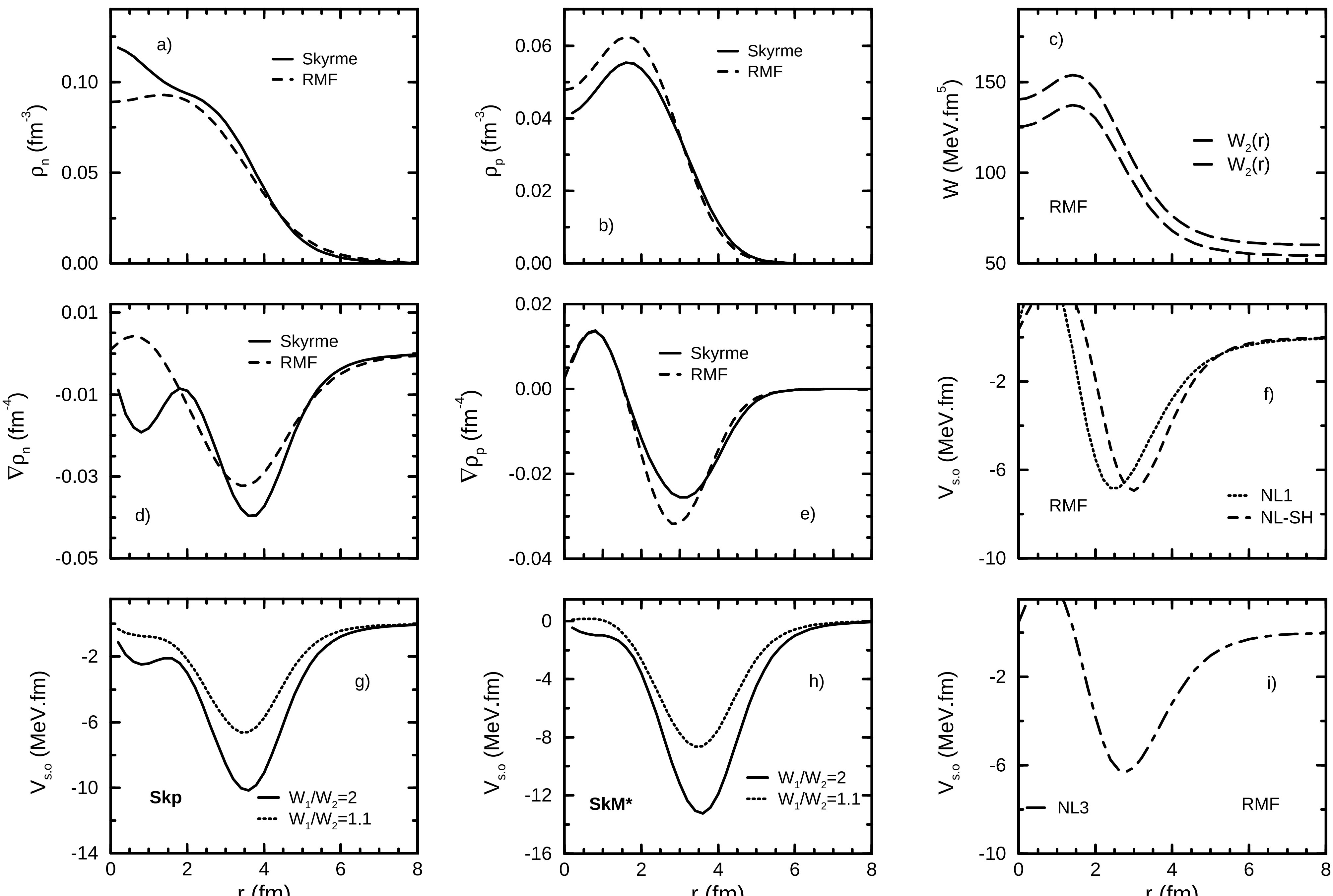


Fig. 2



$1p_{1/2}$-$1p_{3/2}$
$1d_{3/2}$-$1d_{5/2}$
$1f_{5/2}$-$1f_{7/2}$
$2p_{1/2}$-$2p_{3/2}$
E_{ls} [MeV]
N
Ne nuclei
Mg nuclei

Fig. 3



a)
Skyrme
RMF
ρ_n (fm^{-3})
b)
Skyrme
RMF
ρ_p (fm^{-3})
c)
$W_2(r)$
$W_2(r)$
RMF
W (MeV.fm^5)
d)
Skyrme
RMF
$\nabla\rho_n$ (fm^{-4})
e)
Skyrme
RMF
$\nabla\rho_p$ (fm^{-4})
f)
RMF
NL1
NL-SH
$V_{s.o}$ (MeV.fm)
g)
Skp
W_1/W_2=2
W_1/W_2=1.1
$V_{s.o}$ (MeV.fm)
h)
SkM*
W_1/W_2=2
W_1/W_2=1.1
$V_{s.o}$ (MeV.fm)
i)
NL3
RMF
$V_{s.o}$ (MeV.fm)
r (fm)
r (fm)
r (fm)

Fig. 4